Answer Sheet to the Written Exam

Corporate Finance and Incentives

February 2018

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student should be able to:

Knowledge:

1. Understand, account for, define and identify the main methodologies, concepts and topics in Finance
2. Solve standard problems in Finance, partly using Excel
3. Criticize and discuss the main models in Finance, relating them to current issues in financial markets and corporate finance

Skills:

1. Manage the main topics and models in Finance
2. Organize material and analyze given problems, assessing standard models and results
3. Argue about financial topics, putting results into perspective, drawing on the relevant knowledge of the field

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems, and on given descriptions of situations in financial markets or corporations
2. Be prepared for more advanced models and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1 and 2, skills of type 1 and 2, competencies 1 and 2. Problem 4 emphasizes knowledge points 1 and 3, skills 1 and 3, and competency 1.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (CAPM 25%)

1) Use matrix inversion in Excel. Compute $z = A^{-1}\mathbf{1} = (8.30, 24.84, -0.21, 20.38)^T$, normalized to the minimum-variance portfolio $x_m = (0.16, 0.47, -0.00, 0.38)^T$. The expected return is $x_m^T b = 2.13\%$ and the variance is $x_m^T A x_m = 0.0188$.

2) Compute $z = A^{-1}(b - r_f\mathbf{1}) = (0.14, 0.36, 0.01, 0.35)^T$, normalized to tangent portfolio $x_e = (0.16, 0.42, 0.01, 0.41)^T$. Expected return $x_e^T b = 2.51\%$, variance $x_e^T A x_e = 0.0231$.

3) The equally weighted portfolio $x = (0.25, 0.25, 0.25, 0.25)$ has expected return is $x^T b = 3.48\%$ and the variance is $x^T A x = 0.1988$. According to the Two Mutual Fund Theorem, the efficient frontier is traced by convex combinations of our two efficient portfolios from 1) and 2), with positive weight y on the portfolio from 2). To match the expected return, y must solve $3.48 = y2.51 + (1-y)\,2.13$, so the candidate is $y = 3.57$. The convex combined portfolio is $x_c = yx_e + (1-y)\,x_m = (0.19, 0.29, 0.06, 0.47)^T$. Its variance is $x_c^T A x_c = 0.0738$, which is less than $x^T A x$. It follows that the equally weighted portfolio is inefficient.

4) The relevant CAPM equation is $E[R_i] = r_f + \beta_i\,(E[R_{eff}] - r_f)$. We can isolate $\beta_i = (b_i - r_f) / \left(x_e^T b - r_f\right)$. This gives $\beta_1 = 1.89$, $\beta_2 = -1.19$, $\beta_3 = 2.39$, $\beta_4 = 2.84$.

Problem 2 (Merger 25%)

1), 2) and 3) In an outcome where a firm earns $X \geq 0$, its corporate tax payment is $30\%X$ and its legal costs are zero. When it earns $X < 0$, its tax is 0; legal costs are 10 for a stand-alone firm, but 30 for a merged firm. The risk-neutral expectation takes an average of the numbers for the four outcomes, using the probabilities as weights. Results:

	F 1 tax	F 1 cost	F 2 tax	F 2 cost	Merged tax	Merged cost
Outcome 1	15	0	6	0	21	0
Outcome 2	15	0	0	10	12	0
Outcome 3	0	10	6	0	0	30
Outcome 4	0	10	0	10	0	30
Expectation	10.5	3	4.2	3	12.9	9

4) The merged firm's expected tax payment is 12.9, while the sum of stand-alone firms' expected tax payments is $10.5 + 4.2 = 14.7$. This is a general result, actually true outcome by outcome, basically due to the convexity of the tax function $.3\max\{0, X\}$.

The merged firm's expected legal costs are 9 while the sum of stand-alone firms' expected legal costs is $3+3 = 6$. In outcome 2, the merged firm avoids bankruptcy, while firm 2 would have been bankrupt on its own. On the other hand, in outcome 3, the merged firm is bankrupt, while firm 2 would have survived on its own. So outcomes 2 and 3 pull in opposite directions. More importantly, it has been assumed that the merged firm has rather large legal costs in case of bankruptcy, explaining the bottom line.

Looking at the sum of expected tax payments and legal costs, the merged firm has $12.9 + 9 = 21.9$, while the two independent firms sum to $14.7 + 6 = 20.7$ — slightly lower sum, driven by the lower legal costs.

Problem 3 (Options 25%)

1) C is the current market price (or premium) on the call option. P is the price on the put. S is the current market value of the underlying asset. In this formula, PV means present value, i.e., the current market value of a claim on the cash-flow in the parenthesis. With dividends, this means the risky cash-flow paid to a holder of the underlying asset until the option expires. With K, the meaning is a safe payment of K on the expiration date.

2) See Section 20.3 in Berk and DeMarzo. Figure 20.7 or something similar is very helpful.

3) The safe payment of $K = \$2840$ three months ahead should be discounted by the safe interest rate. This is annually 1.44%. The result is

$$PV\left(K\right) = \frac{\$2840}{1.0144^{3/12}} = \$2829.87.$$

4) Rearranging the put-call parity, we can isolate $PV\left(Div\right) = P + S - C - PV\left(K\right) = \$59.10 + \$2845.20 - \$64.30 - \$2829.87 = \$10.13$.

Problem 4 (Various Themes 25%)

1) See Section 3.2.2 in the lecture notes. p_i is the current market price of any asset i, r is a risk-free interest rate, q_j is the risk-neutral probability of state j, and v_{ij} is the future value of asset i in state j. The absence of arbitrage implies the existence of these risk-neutral probabilities that explain asset prices in this fashion, but q_j need not be the probability attached by any trader to the occurrence of state j.

2) Section 31.3 in Berk and DeMarzo discusses repatriation of earnings. Firms may seek to reduce their overall tax bill by deferring repatriation, as Apple appears to have done. If Apple's corporate tax rate is now lower than before, its WACC should be lower, and so it should be interested in financing more projects than before. The text implicitly suggests that Apple's true tax rate, and hence its WACC in international capital markets, may really be unchanged. The repatriated earnings may not be needed for any investment by Apple, and could instead be paid out at a tax-favorable time as a dividend to its shareholders.

3) Section 16.5 in Berk and DeMarzo discusses the asset substitution problem as the result of a conflict among equity holders and debt holders in a firm. Equity holders have an incentive to increase risk. See also the end of Section 21.5, building on the interpretation of equity as a call option from Section 20.6. Chapter 29 is focused on conflicts between firms' managers and investors, but the ideas can be applied to the conflict among equity holders and debt holders, viewing debt holders as the principal and equity holders as the agent.